



# Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level In  
Decision Mathematics D1 (WDM11/01)

| Question Number | Scheme   | Marks                              |
|-----------------|--|------------------------------------|
| <b>1. (a)</b>   | Nearest neighbour: A – B – F – D – E – C – A<br>35 31 44 39 53 42 = 244 (km) | M1<br>A1 <b>(2)</b>                |
| <b>(b)</b>      | MST with B removed: AC, CD, DF, DE gives a RMST weight of 172 (km)           | B1                                 |
|                 | 172 + 31 + 35 = 238 (km)   | M1 A1 <b>(3)</b><br><b>5 marks</b> |
|                 | <b>Notes for Question 1</b>  |                                    |

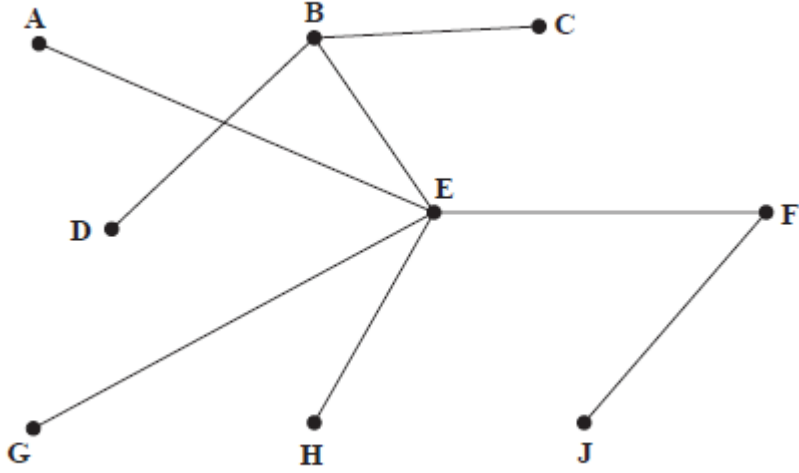
**a1M1:** Nearest neighbour A – B – F – D – E – C – (condone lack of return to start) **or** correct route length of 244. Accept AB, BF, FD, DE, EC but do not accept weights only. Accept 1 2 6 4 5 3 across the top of the table

**a1A1:** CAO both route (either in terms of vertices (ABFDECA) or arcs (AB, BF, FD, DE, EC, CA) but not weights) **and** length correct (244) do not ISW if this value is then doubled to 488

**b1B1:** CAO for RMST weight (either 172 or 42 + 47 + 44 + 39) – maybe implied by later working

**b1M1:** Adding 31 + 35 (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs

**b1A1:** CAO - if 238 seen without working then award B0M1A1

| Question Number | Scheme   | Marks            |
|-----------------|--|------------------|
| 2. (a)(i)       | A tree is a connected graph with no cycles   | B1               |
| (ii)            | A minimum spanning tree is a tree that contains all vertices<br>The total length of its arcs is as small as possible                       | B1<br>B1 (3)     |
| (b)             | Kruskal: FJ(11), EG(13), EF(15), EH(17), not GH (18), BC (19), not HJ (20), BD (22), not FH(23), AE (25), BE (29) (not AD, DE, DG, AB, BH) | M1 A1 A1<br>(3)  |
| (c)             |  <p>(Weight of the tree is) 151</p>                      | B1<br><br>B1 (2) |
|                 |  | 8 marks          |

### Notes for Question 2

**ai1B1: Connected + no cycle(s)** (must contain these two points – do not allow ‘circle’, ‘loop’ etc. for cycle(s)) – if not using the word ‘connected’ then allow ‘a graph that connects the vertices/nodes’ (condone issues with plural or singular e.g. cycle for cycles)

**aii2B1: Contains all vertices/nodes** (must be clear that all vertices (or nodes) are in a MST)

**aii3B1: Total length of arcs is minimised** (must contain the three points regarding weight/length, arcs/edges and minimised/smallest (oe))

**b1M1: Kruskal’s algorithm** - first four arcs (FJ, EG, EF, EH) correctly chosen **and** at least one rejection seen at some point

**b1A1: All arcs in tree selected correctly in the correct order** (FJ, EG, EF, EH, BC, BD, AE, BE) with no additional arcs included in MST

**b2A1: CSO including all rejections correct and at the correct time** (do not need to see AD, DE, DG, AB, BH rejected but if they are they must be rejected correctly (i.e. in this order) but note that AD, DE have the same weight as do DG and AB so they could appear in either order)

Note that stating all the arcs in order (e.g. FJ, EG, EF, EH, GH, BC, HJ, BD, FH, AE, BE, AD, DE, DG, AB, BH) and then stating only those in the tree in the correct order is fine for all three marks in this part

**c1B1: CAO (tree)**

**c2B1: CAO (151)**

| Question Number | Scheme   | Marks                           |
|-----------------|--|---------------------------------|
| <b>3.(a)</b>    | $x = 8, y = 12, z = 17$  | B3, 2, 1, 0 <b>(3)</b>          |
| <b>(b)</b>      |  | M1<br>A1<br>A1<br>A1 <b>(4)</b> |
| <b>(c)</b>      | Lower bound is 4 workers e.g. activities J, K, L and M together with $24 < \text{time} < 26$ | M1 A1 <b>(2)</b>                |
|                 |  | <b>9 marks</b>                  |

### Notes for Question 3

**a1B1:**  $x$  value correct

**a2B1:**  $y$  value correct

**a3B1:**  $z$  value correct

**b1M1:** At least ten activities labelled including at least five floats. A scheduling diagram scores M0

**b1A1:** The critical activities dealt with correctly and appearing just once (C, H, I and J) and three non-critical activities dealt with correctly

**b2A1:** Any six non-critical activities correct (this mark is not dependent on the previous A mark)

**b3A1:** CSO – completely correct Gantt chart (exactly thirteen activities appearing just once)

**c1M1:** Either a statement with the correct number of workers (4) and the correct activities (J, K, L and M) with any numerical time stated **or** the correct number of workers (4) and a time in the interval  $24 \leq x \leq 26$  – mark the numerical value only not their use of day/time

**c1A1:** A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of  $24 < t < 26$ . Please note the strict inequalities for the time interval (e.g. implying a time of 24 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 25 – 26 is A0). Note that ‘on day 25’ or ‘on day 26’ are correct but ‘on day 24’ is not correct. A completely correct statement with an additional incorrect statement scores A0 (so no isw)

| Question Number | Scheme   | Marks                         |
|-----------------|--|-------------------------------|
| 4. (a)          | Bin 1: <b>35</b> 17 7<br>Bin 2: <b>10</b> <u>28</u> <u>15</u><br>Bin 3: <u>23</u> 20<br>Bin 4: <u>41</u><br>Bin 5: 29  | <b>M1</b> <u>A1</u> A1<br>(3) |
| (b)             | e.g. middle right Pivot(s)   |                               |
|                 | 35 17 10 7 28 <u>23</u> 41 15 20 29 23<br>35 28 <u>41</u> 29 <b>23</b> 17 10 <u>7</u> 15 20 41, 7<br><b>41</b> 35 <u>28</u> 29 <b>23</b> 17 10 <u>15</u> 20 7 28, 15<br><b>41</b> 35 <u>29</u> <b>28</b> <b>23</b> 17 <u>20</u> <b>15</b> 10 7 29, 20, (10)<br><b>41</b> 35 <b>29</b> <b>28</b> <b>23</b> <b>20</b> 17 <b>15</b> <b>10</b> 7 Sort complete | M1<br>A1<br>A1ft<br>A1<br>(4) |
| (c)             | Bin 1: <b>41</b> 17<br>Bin 2: <b>35</b> <b>23</b><br>Bin 3: <b>29</b> <b>28</b><br>Bin 4: <u>20</u> 15 10 7  | <b>M1</b> <u>A1</u> A1<br>(3) |
| (d)             | $8 < x < 12, y > x, (y > 8)$   | B3, 2, 1, 0 (3)               |
|                 |  | <b>13 marks</b>               |

#### Notes for Question 4

**PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASES IN PART (b) and the guidance for the M mark in (c)**

**a1M1:** First four items placed correctly (the values in bold) and at least seven values placed in bins.

Condone cumulative totals for M1 only – if one of the bold values appears in more than one bin then M0

**a1A1:** First eight items placed correctly (the underlined and bold values) – if one of the underlined values appears in two different bins then this is A0

**a2A1:** CSO (correct solution only – so no additional/repeated values)

**b1M1:** Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only**

**b1A1:** First two passes correct

**b2A1ft:** Third pass correct (follow through from their second pass and choice of pivots for the third pass (these pivots for the third pass must be either middle left or middle right)

**b3A1:** CSO (correct solution only – all previous marks in this part **must** have been awarded) including a ‘sort complete’ - this could be shown by the final list being re-written or ‘sorted’ statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

middle left

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35  17  10  7  28 23  41  15  20  29      28
35  41 29  28 17  10  7  23  15  20      41, 7
41 35 29  28 17  10  23 15  20  7      35, 23
41 35 29  28 23 17  10 15  20  7      (29), 10
41 35 29  28 23 17  15 20  10 7      15
41 35 29  28 23 17 20  15 10 7      17
41 35 29  28 23 20  17  15 10 7      Sort complete

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| Question Number   | Scheme | Marks |
|---|--------|-------|
| <p><b>SC for (b): If using an incorrect list from the start of (b) with only one error (an error is either one missing number, one extra number, two numbers transposed or one incorrect number) then the most they can score is M1A0A1ftA0</b></p> <p><b>Sorting list into ascending order in (b)</b></p> <ul style="list-style-type: none"> <li>• If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in (b)</li> <li>• If the list is not reversed in (b) then remove the last two A marks earned in (b). If the list is reversed at the start of (c) but not in (b) then still remove the last two A marks earned in (b). If the list is in ascending order in (b) award no marks for first-fit increasing in (c). If the candidate says that the list needs reversing in (b) but does not actually show the reversed list in (b) then remove the last A mark earned</li> </ul> <p><b>c1M1: Must be using a list</b> that is in strictly descending order. If it is clear that their list is not in descending order then M0. First five items placed correctly (the bold values) and at least eight values placed in bins – condone cumulative totals for M1 only – if one of the bold values appears in more than one bin then M0. If it is clear that their list is not correct (41 35 29 28 23 20 17 15 10 7) then M1 only and for this M mark allow them to be using their final list from (b) which can contain one error and in this case an error is either one missing number or one extra number or one incorrect number (e.g. 24 for 23) but their list must be in descending order.</p> <p><b>c1A1:</b> First seven items placed correctly (the underlined and bold values) – if one of the underlined values appears in two different bins then this is A0</p> <p><b>c2A1:</b> CSO (so no additional/repeated values)</p> <p><b>d1B1:</b> <math>x &gt; 8</math></p> <p><b>d2B1:</b> <math>x &lt; 12</math></p> <p>If B0 B0 then award B1 B0 for <math>8 \leq x \leq 12</math> (oe)</p> <p><b>d3B1:</b> <math>x &lt; y</math> but not <math>y &gt; 8</math> only</p> <p>For full marks in (d) no additional incorrect constraints, for example, if B1B1B1 initially awarded but an additional incorrect constraint seen (e.g. <math>y &gt; 15</math>) then award B1B1B0 but do not penalise any additional incorrect constraints unless all three other correct constraints seen</p> |        |       |

| Question Number | Scheme                                      | Marks                                    |
|-----------------|---|--|
| 5. (a)          |   | M1<br>A1<br>A1<br>A1<br>A1<br><b>(5)</b> |
| (b)             | Activities E, I, J and L cannot be critical | M1 A1 <b>(2)</b>                         |
|                 |   | <b>7 marks</b>                           |

#### Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. The table below is very useful in checking this point. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be) '

#### Ignore incorrect or lack of arrows on the activities for the first four marks only

**a1M1:** Seven activities (labelled on arc), one start and at least two dummies placed

**a1A1:** Activities A, B, C, 1<sup>st</sup> two dummies (including correct arrows on these two dummies), D and E dealt with correctly. The first two dummies are those at the end of activities A and C

**a2A1:** A dummy linking D, F and G into an event node and a dummy linking D, F, G, I and E into an event node and F, G and I dealt with correctly. The dummies must have arrows

**a3A1:** Activities H, J, K and L dealt with correctly

**a4A1:** CSO – all arrows correctly placed for each activity with one finish and at most four dummies. Note that this is not a unique solution due to for example the 3<sup>rd</sup> and 4<sup>th</sup> dummies so please check these carefully.

**Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.**

**Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned**

**b1M1:** At least two correct activities with no more than 5 activities stated

**b1A1:** All four activities (E, I, J and L) and no others

| A | B | C | D | E | F       | G       | H       | I       | J       | K | L             |
|---|---|---|---|---|---------|---------|---------|---------|---------|---|---------------|
| - | - | - | A | C | A, B, C | A, B, C | D, F, G | A, B, C | D, F, G | H | D, E, F, G, I |

| Question Number | Scheme   | Marks  |
|-----------------|--|--|
| 6. (a)          | <p>Shortest time: 45 (minutes)<br/>Quickest route: A C B D F G J</p>   | <p>M1</p> <p>A1 (CEBD)</p> <p>A1 (FH)</p> <p>A1ft (GJ)</p> <p>A1ft</p> <p>A1 (6)</p> |
| (b)             | $A(CB)D + HJ = 20 + 20 = 40$<br>$A(CBDF)H + D(FG)J = 26 + 25 = 51$<br>$A(CBDFG)J + D(F)H = 45 + 6 = 51$<br>Repeated arcs: AC, BC, BD, HJ   | <p>M1 A1</p> <p>A1</p> <p>A1</p> <p>A1 (5)</p>                                       |
| (c)(i)          | Vertex C: 4 times  | B1   |
| (ii)            | Vertex D: 3 times  | B1 (2)   |
| (d)             | H to H requires the consideration of the shortest path from A to J only (as these are the only two odd nodes)<br>$314 > 309$ or $45 > 40$ so quicker to start at H and finish at D | <p>B1</p> <p>B1 (2)</p>  |
|                 |  | 15 marks   |

#### Notes for Question 6

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at J the working values must be 49 46 45 in that order (so 49 45 46 is incorrect)

It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

**a1M1:** A larger value replaced by a smaller value in at least two of the working value boxes at any node except A or C

**a1A1:** All values at C, E, B and D correct and the working values in the correct order (including order of labelling)

**a2A1:** All values at F and H correct and the working values in the correct order. Penalise order of labelling only once per question (F and H must be labelled in that order and F must be labelled after C, E, B and D)

**a3A1ft:** All values in G and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will mostly likely come from nodes B, D and F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from E, H and G with the order of these values determined by the candidate's order of labelling at E, H and G)



| Question Number | Scheme   | Marks |
|-----------------|--|-------|
|                 | <p><b>a4A1ft:</b> Follow through on their final value at J <b>only</b> (condone lack of units) - so if 45 given as the answer and the final value at J is not 45 then A0</p> <p><b>a5A1:</b> CAO - correct route (from either A to J or J to A) – ACBDFGJ or JGFDBCA</p> <p><b>b1M1:</b> Correct three distinct pairings of the correct four odd nodes A, D, H and J</p> <p><b>b1A1:</b> Any row correct including pairing <b>and</b> total</p> <p><b>b2A1:</b> Any two rows correct including pairings <b>and</b> totals</p> <p><b>b3A1:</b> All three rows correct including pairings <b>and</b> totals</p> <p><b>b4A1:</b> CAO correct edges clearly stated and not just in their working as AC, BC, BD, HJ. Must be these arcs and not AD, ACBD or AD via B and C</p> <p><b>ciB1:</b> CAO (Vertex C: 4)</p> <p><b>ciiB1:</b> CAO (Vertex D: 3)</p> <p><b>d1B1:</b> Correct reasoning (that to travel from H to H) only the shortest path between A and J needs traversing twice – as a minimum must mention either ‘A to J <u>only</u>’ or refer to A and J being the only odd nodes (e.g. odd nodes: A and J is fine but not ‘A and J are odd nodes’)</p> <p><b>d2B1:</b> Either ‘it will be slower’ or ‘it will be quicker from H to D’ (if saying ‘quicker’ then it must be clear that they are talking about H to D) + correct numerical <u>argument</u> (not just stating the values 314 and 309 (or 45 and 40) and saying ‘slower’ - there must be some comparison of these two values )</p> |       |

| Question Number | Scheme   | Marks                                 |
|-----------------|--|---------------------------------------|
| 7. (a)          | Minimise $400x + 550y + 750z$<br>Subject to $x + y + z \geq 20$<br>$x \leq 2(y + z)$<br>$z \leq \frac{1}{4}(x + y + z) \Rightarrow 3z \leq x + y$  | B1<br>B1<br>M1 A1<br>M1 A1 <b>(6)</b> |
| (b)(i)(ii)      | Ratio of 5:3 $\Rightarrow 3y = 5z$<br>$x + y + z \geq 20 \Rightarrow x + y + \frac{3}{5}y \geq 20$ and $400x + 550y + 750\left(\frac{3y}{5}\right)$<br>$5x + 8y \geq 100$ and $400x + 1000y$ | B1<br>M1<br>A1 <b>(3)</b>             |
| (c)             |  | B1<br>B1<br>B1<br>B1 <b>(4)</b>       |
| (d)             | Drawing an objective line accept reciprocal gradient<br>Correct objective line<br>V correctly labelled   | M1<br>A1<br>A1 <b>(3)</b>             |
| (e)(i)<br>(ii)  | Cost is £9800<br>3 days for senior leaders (12 training days for new teachers, 5 days for middle leaders)  | B1<br>B1 <b>(2)</b>                   |
|                 |  | <b>18 marks</b>                       |

| Question Number  | Scheme | Marks |
|--|--------|-------|
| <b>Notes for Question 7</b>  |        |       |
| <p><b>a1B1:</b> Expression correct <u>together</u> with ‘minimise’ or ‘min’ but not ‘minimum’ – isw if coefficients are subsequently simplified but <math>400x + 550y + 750z</math> must be seen at some point for this mark to be awarded. The ‘min’ must appear beside the correct expression</p> <p><b>a2B1:</b> CAO <math>(x + y + z \geq 20)</math></p> <p><b>a1M1:</b> <math>x \square 2(y + z)</math> where <math>\square</math> is any inequality or equals. Accept <math>2x \leq y + z</math> for this mark</p> <p><b>a1A1:</b> CAO <math>x \leq 2(y + z)</math> or equivalent but only one term in each variable and integer coefficients</p> <p><b>a2M1:</b> <math>z \square \frac{1}{4}(x + y + z)</math> where <math>\square</math> is any inequality or equals – allow 0.25 but do not allow <math>z \square 25\%(x + y + z)</math> unless recovered to a fraction or decimal later</p> <p><b>a2A1:</b> CAO <math>3z \leq x + y</math> or equivalent but only one term in each variable and integer coefficients</p> <p><b>bi1B1:</b> Ratio expressed correctly as an equation <math>3y = 5z</math> (oe) (possibly implied by subsequent working)</p> <p><b>bi1M1:</b> Their linear equation in <math>y</math> and <math>z</math> only (must be of the form <math>ay = bz</math> oe) substituted into their constraint and their objective to eliminate <math>z</math> <b>or</b> one correct answer simplified</p> <p><b>b1A1:</b> CAO <math>5x + 8y \geq 100</math> and <math>400x + 1000y</math> (allow ISW if correct objective simplified)</p> <p>In (c)</p> <p><math>5x + 8y = 100</math> must pass if extended within one small square of its intersection with the axes – (0, 12.5) and (20, 0) but must be long enough to define the correct feasible region</p> <p><math>4y = 5x</math> must pass within one small square of its intersection with the origin and (10, 12.5)</p> <p><math>5x = 16y</math> must pass within one small square of its intersection with the origin and (20, 6.25)</p> <p><b>c1B1:</b> One line correct</p> <p><b>c2B1:</b> Two lines correct</p> <p><b>c3B1:</b> All three lines correct</p> <p><b>c4B1:</b> Region, <math>R</math>, correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part</p> <p><b>d1M1:</b> Drawing their objective line (based on their answer to (b)) or its reciprocal - line must be correct to within one small square if extended from axis to axis. If their line is shorter than (0, 1) to (2.5, 0) then M0</p> <p><b>d1A1:</b> Drawing a <b>correct</b> objective line subject to the conditions above regarding length and if extended from axis to axis</p> <p><b>d2A1:</b> <math>V</math> labelled clearly on their graph (so dependent on the first three marks in (c) and the previous A mark in this part)</p> <p><b>eiB1:</b> CAO – condone lack of units – dependent on the first three marks in (c) and the first two marks in (d)</p> <p><b>eiiB1:</b> CAO (3) – dependent on the first three marks in (c) and the first two marks in (d)</p> |        |       |